Basic Classes of Mathematical Models

with sample equations

Mathematical model Linear model

$$egin{aligned} oldsymbol{y} &= A oldsymbol{x} + B \ oldsymbol{dy} &= A \ oldsymbol{dx} \end{aligned}$$

Kind of change

Rate of change is constant.

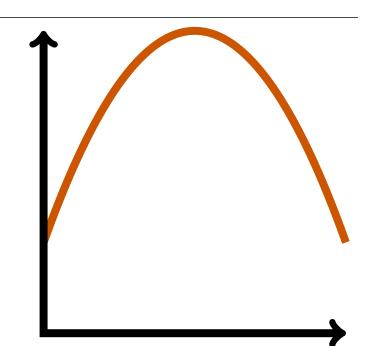


Quadratic model

$$\frac{\mathbf{y} = A\mathbf{x}^2 + B\mathbf{x} + C}{d^2\mathbf{y}} = A$$

$$\frac{d^2\mathbf{y}}{d\mathbf{x}^2} = A$$

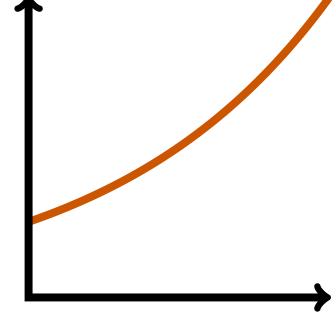
Rate of change of rate of change is constant.



Exponential model

$$rac{oldsymbol{y}}{doldsymbol{y}} = Ab^{oldsymbol{x}} ext{ or } oldsymbol{y} = Ae^{rac{oldsymbol{x}}{\xi}} \ rac{doldsymbol{y}}{doldsymbol{x}} = \ln b \cdot oldsymbol{y} ext{ or } rac{doldsymbol{y}}{doldsymbol{x}} = rac{oldsymbol{y}}{\xi}$$

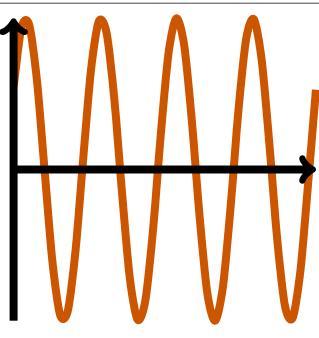
Rate of change is proportional to amount.



Harmonic model

$$egin{aligned} oldsymbol{y} &= A\cos\left(koldsymbol{x} + \phi
ight) ext{ or } \ oldsymbol{y} &= A\sin\left(koldsymbol{x} + \phi'
ight) ext{ or } \ oldsymbol{y} &= \mathfrak{Re}\left\{Ae^{i(koldsymbol{x} + \phi)}
ight\} \ rac{d^2oldsymbol{y}}{doldsymbol{x}^2} &= -k^2oldsymbol{y} \end{aligned}$$

Rate of change of rate of change is proportional to amount.



Sudden change model Change is finite

$$egin{aligned} oldsymbol{y} &= A \ heta(oldsymbol{x} - x_0) + B \ rac{doldsymbol{y}}{doldsymbol{x}} &= A \ \delta(oldsymbol{x} - x_0) \ rac{doldsymbol{x}}{doldsymbol{x}} \end{aligned}$$

Change is finite and and instantaneous.

